CAMBRIDGE
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FURTHER MATHEMATICS **9231/24**

Paper 2 Further Pure Mathematics 2 **May/June 2025**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

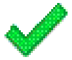
We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
✗	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	**Meaning**
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
~~	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\begin{vmatrix} 1 & 2 & 3 \\ k & 5 & 6 \\ 7 & 2k & 9 \end{vmatrix} = 45 - 12k - 2(9k - 42) + 3(2k^2 - 35)$	**M1**	Expands determinant and forms quadratic expression.
	$6k^2 - 30k + 24 = 0 \Rightarrow k = 1, 4$	**M1 A1**	Solves quadratic. Need both values for A1.
		3	
1(b)	$\begin{array}{l} x + 2y + 3z = 1, \\ x + 5y + 6z = 2, \\ 7x + 2y + 9z = 3, \end{array} \Rightarrow \begin{array}{l} 3y + 3z = 1 \\ 3x + 3z = 1 \end{array}$	**M1 A1**	For M1 must use all three equations and reduce to e.g. $y = x$. Accept explicit solution using a parameter. Accept row echelon form (with clear operations on all three rows) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.
	Three planes meeting in a common line.	**B1**	Sheaf.
		3	

Question	Answer	Marks	Guidance
2	$x^2 - 2x + 5 = 4 + (x-1)^2$	**B1**	Completes the square. OE.
	$\int_1^{\frac{5}{2}} \frac{1}{\sqrt{4+(x-1)^2}}\,dx = \left[\sinh^{-1}\left(\frac{x-1}{2}\right)\right]_1^{\frac{5}{2}}$	**M1 A1**	Applies formula.
	$= \sinh^{-1}\left(\frac{3}{4}\right) - \sinh^{-1}(0)$	**M1**	Uses limits.
	$= \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right) = \ln 2$	**M1 A1**	Uses logarithmic form.
		6	

Question	Answer	Marks	Guidance
3	$m^2+4m+5=0 \Rightarrow m=-2\pm\mathrm{i}$	**M1**	Auxiliary equation. M1 for correct type of roots.
	$y=\mathrm{e}^{-2x}(A\cos x+B\sin x)$	**A1**	Complementary function. Allow '$y=$' missing. Accept $y=\mathrm{e}^{-2x}(A\cos x+B\mathrm{i}\sin x)$.
	$y=k\mathrm{e}^{3x} \Rightarrow y'=3k\mathrm{e}^{3x} \Rightarrow y''=9k\mathrm{e}^{3x}$	**B1**	Particular integral and its derivatives.
	$9k+12k+5k=13 \Rightarrow k=\frac{1}{2}$	**M1 A1**	Substitutes and cancels e^{3x}.
	$y=\mathrm{e}^{-2x}(A\cos x+B\sin x)+\frac{1}{2}\mathrm{e}^{3x}$	**A1 FT**	General solution. Must have '$y=$'. FT on *their* CF (The PI must be correct).
	$A+k=1 \Rightarrow A=1-k=\frac{1}{2}$	**B1 FT**	Uses $y(0)=1$, FT on their $1-k$. Their CF must be correct.
	$y'=\mathrm{e}^{-2x}(-A\sin x+B\cos x)-2\mathrm{e}^{-2x}(A\cos x+B\sin x)+\frac{3}{2}\mathrm{e}^{3x}$	**M1**	Finds y'. For M1, their CF must have the correct form.
	$B-2A+\frac{3}{2}=0 \Rightarrow B=-\frac{1}{2}$	**A1**	Uses $y'(0)=0$.
	$y=\frac{1}{2}\mathrm{e}^{-2x}(\cos x-\sin x)+\frac{1}{2}\mathrm{e}^{3x}$	**A1**	Must have '$y=$'.
		10	

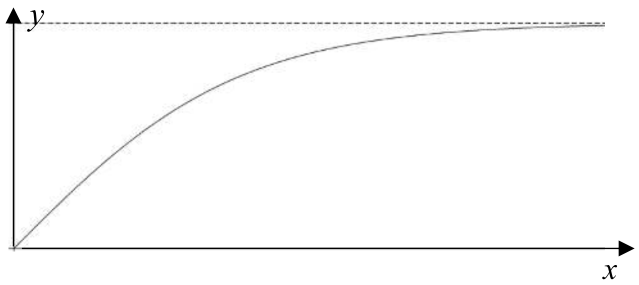
Question	Answer	Marks	Guidance
4(a)	$x=0 \Rightarrow (t-1)(t^2+1)=0$	**B1**	Factorises or considers other roots of $x=0$.
		1	

Question	Answer	Marks	Guidance
4(b)	$\frac{dx}{dt} = 3t^2 - 2t + 1 \quad \frac{dy}{dt} = (t+1)e^t$	B1	
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{(t+1)e^t}{3t^2 - 2t + 1}$	M1 A1	Applies chain rule, AG.
		3	
4(c)	$\frac{d}{dt}\left(\frac{(t+1)e^t}{3t^2 - 2t + 1}\right) = \frac{\left(3t^2 - 2t + 1\right)\left((t+1)e^t + e^t\right) - (t+1)e^t\left(6t - 2\right)}{\left(3t^2 - 2t + 1\right)^2}$	*M1	Differentiates $\frac{dy}{dx}$ with respect to t.
	$\frac{d^2y}{dx^2} = \frac{\left(3t^2 - 2t + 1\right)(t+2)e^t - (t+1)\left(6t - 2\right)e^t}{\left(3t^2 - 2t + 1\right)^3}$	M1 A1	Applies chain rule.
	$y(0) = e, \quad y'(0) = e, \quad y''(0) = -\frac{1}{4}e$	A1	Evaluates derivatives at $t = 1$.
	$y = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 = e + ex - \frac{1}{8}ex^2$	DM1 A1	Finds Maclaurin's series. For M1, must have attempted second derivative.
		6	

Question	Answer	Marks	Guidance
5(a)	$\sin 7\theta = \mathrm{Im}\left((c+\mathrm{i}s)^7\right)$	**M1**	Uses binomial expansion and takes imaginary part.
	$= -s^7 + 21c^2s^5 - 35c^4s^3 + 7c^6s$	**A1**	
	$= -s^7 + 21(1-s^2)s^5 - 35(1-s^2)^2 s^3 + 7(1-s^2)^3 s$	**M1**	Applies $c^2 = 1 - s^2$.
	$(1-s^2)^3 = -s^6 + 3s^4 - 3s^2 + 1$	**B1**	No sight of $(1-s^2)^3$ *fully* expanded is B0.
	$\sin 7\theta = -64s^7 + 112s^5 - 56s^3 + 7s$	**A1**	AG.
		5	
5(b)	$x = \sin\theta \quad \sin\theta \neq 0 \quad \sin 7\theta = 0$	**M1**	Solves $\sin 7\theta = 0$ and excludes $\sin\theta = 0$. For M1 the roots from $\sin\theta = 0$ must be excluded at some stage of their working. (This could be implied by their final answers.)
	$x = \sin\left(\frac{1}{7}\pi\right)$	**A1**	One correct root.
	$\sin\left(-\frac{1}{7}\pi\right), \quad \sin\left(\pm\frac{2}{7}\pi\right), \quad \sin\left(\pm\frac{3}{7}\pi\right)$	**A1**	Exactly five other correct roots. Allow different values of q which give these roots. Accept $-\sin\left(\frac{1}{7}\pi\right), \quad \pm\sin\left(\frac{2}{7}\pi\right), \quad \pm\sin\left(\frac{3}{7}\pi\right)$. Just listing correct values of q without any reference of x is M1 SCB1 (2/3).
		3	

Question	Answer	Marks	Guidance
6	$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{x}y = 2x\tan^{-1}x$	B1	Divides through by x.
	$\mathrm{e}^{-\int x^{-1}\,\mathrm{d}x} = x^{-1}$	M1 A1	Finds integrating factor.
	$\frac{\mathrm{d}}{\mathrm{d}x}(yx^{-1}) = 2\tan^{-1}x$	*M1	Correct form on LHS and attempt to integrate RHS by parts. Treating $\tan^{-1}x$ as $\cot x$ is M0.
	$yx^{-1} = 2x\tan^{-1}x - 2\int \frac{x}{1+x^2}\,\mathrm{d}x = 2x\tan^{-1}x - \ln\left(1+x^2\right) + C$	A1 A1	$-\ln\left(1+x^2\right) = \ln\cos\left(\tan^{-1}x\right)$. SC B1 for $x\tan^{-1}x - \frac{1}{2}\ln\left(1+x^2\right) + C$
	$\frac{1}{2}\pi = 2\left(\frac{1}{4}\pi\right) - \ln 2 + C \Rightarrow C = \ln 2$	DM1	Finds C, substitutes given conditions into their solution.
	$y = 2x^2\tan^{-1}x - x\ln\left(1+x^2\right) + x\ln 2$	DM1 A1	Divides through by their coefficient of y.
		9	

Question	Answer	Marks	Guidance
7(a)	Eigenvalues of **A** are $1,\ 2$ and -3.	**B1**	Upper diagonal matrix or characteristic equation.
	$\lambda = 1:\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 5 \\ 0 & 0 & -4 \end{vmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1 A1**	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 2:\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 7 & 11 \\ 0 & 0 & 5 \end{vmatrix} = \begin{pmatrix} 35 \\ 5 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$	**A1**	
	$\lambda = -3:\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 7 & 11 \\ 0 & 5 & 5 \end{vmatrix} = \begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	**A1**	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 7 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 729 \end{pmatrix}$	**M1 A1**	Or correctly matched permutations of columns. OE. A column of zeros or a repeated column is M0.
		7	
7(b)	$(\lambda-1)(\lambda-2)(\lambda+3) = \lambda^3 - 7\lambda + 6 = 0$	**B1**	Characteristic equation. '$=0$' can be implied later.
	$\mathbf{A}^3 = 7\mathbf{A} - 6\mathbf{I}$	**M1**	Substitutes for **A** and makes $\mathbf{A}^3$ the subject. (Could substitute for **A** at the end).
	$\mathbf{A}^6 = (7\mathbf{A} - 6\mathbf{I})(7\mathbf{A} - 6\mathbf{I}) = 49\mathbf{A}^2 - 84\mathbf{A} + 36\mathbf{I}$	**M1 A1**	Expands $(7\mathbf{A} - 6\mathbf{I})(7\mathbf{A} - 6\mathbf{I})$.
		4	

Question	Answer	Marks	Guidance
8(a)		**B1**	Correct shape and position. Ignore $x < 0$ on their sketch.
	$y = 1$	**B1**	States the equation of the asymptote. Ignore $y = -1$.
		2	
8(b)	$\sum_{r=1}^{N} \tanh r > \int_0^N \tanh x \, dx$	**M1 A1**	Compares sum with integral. Limits on integral must be present and correct for M1.
	$\int_0^N \tanh x = \left[\ln(\cosh x)\right]_0^N = \ln(\cosh N)$	**A1**	Evaluates integral, AG. Must see $\ln(\cosh x)$.
		3	

Question	Answer	Marks	Guidance
8(c)(i)	$S = 2\pi\int_0^{\frac{1}{2}\ln 3} y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi\int_0^{\frac{1}{2}\ln 3}\tanh(x)\sqrt{1+\text{sech}^4(x)}\,dx$	M1 A1	Correct formula for surface area. Missing limits is M1 A0. y outside of integral is M0.
	$u=\sqrt{1+\text{sech}^4 x} \Rightarrow \frac{du}{dx} = -2\left(1+\text{sech}^4 x\right)^{-\frac{1}{2}}\text{sech}^4 x\tanh x$	M1 A1	Finds $\frac{du}{dx}$. Sign error gets M1 A0.
	$\left[x=\frac{1}{2}\ln 3 \Rightarrow \cosh x = \frac{2}{3}\sqrt{3}\right]$ $\text{sech}\, x = \frac{1}{2}\sqrt{3} \Rightarrow u = \sqrt{1+\frac{9}{16}}$	M1 A1	Finds u when $x=\frac{1}{2}\ln 3$. Need to see working to justify given limit. Working without evidence of evaluating $\text{sech}\, x$ is M1 A0.
	$S = -\pi\int_{\sqrt{2}}^{\frac{5}{4}} u\frac{u}{u^2-1}\,du = \pi\int_{\frac{5}{4}}^{\sqrt{2}}\frac{u^2}{u^2-1}\,du$	A1	AG, must have gained all previous marks.
		7	
8(c)(ii)	$=\frac{1}{2}\pi\int_{\frac{5}{4}}^{\sqrt{2}} 2+\frac{1}{u-1}-\frac{1}{u+1}\,du = \pi\left[u+\frac{1}{2}\ln(u-1)-\frac{1}{2}\ln(u+1)\right]_{\frac{5}{4}}^{\sqrt{2}}$	M1 A1	Uses partial fractions or $\frac{u^2}{u^2-1} = 1+\frac{1}{u^2-1}$.
	$=\pi\left(\sqrt{2}+\frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)-\frac{5}{4}-\frac{1}{2}\ln\left(\frac{1}{9}\right)\right) = \pi\left(\sqrt{2}-\frac{5}{4}+\ln 3+\frac{1}{2}\ln\left(3-2\sqrt{2}\right)\right)$	A1	OE.
		3	